$$\cos\beta = \frac{2[(r_1/r_E) - 1] + \gamma_E^2}{\{\gamma_E^2 - 3[(r_1/r_E) - 1]^2\}^{1/2}} \tag{22}$$

For general entry angle γ_E and $r_1 \to r_E$, Eq. (12) reduces to

$$\cos\beta = \sin\gamma_E \tag{23}$$

Equation (23) is in agreement with (22); a different result was given in Ref. 1, where $\beta \to 90°$ as $r_1 \to r_E$.

Figure 3 shows the definite advantage of deboosting from the optimum orbital altitude. Operation at any other altitude results in increases in deboost propellant with attendant decreases in payload. The retrofire alignment angle is zero degrees at all altitudes greater than or equal to the optimum. For altitudes less than optimum, β increases from zero degrees up to the value given by Eq. (23).

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Method of Analyzing Laminar Air Arc-Tunnel Heat Transfer Data

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The Fay-Riddell equations with a non-Newtonian velocity gradient are proposed as models for evaluating laminar arc-tunnel heat transfer data. Means are suggested for ways to estimate the necessary parameters. Calculations indicate that flow can remain partially frozen even to pressures of 10 psia. Calorimeter poisoning may produce marked variations in reported effective heats of ablation.

Nomenclature

D = diameter of calorimeter, ft
h = enthalpy, Btu/lbm
H^* = effective heat of ablation, Btu/lbm
Le = Lewis number, $\rho D_{12}\bar{c}_p/k$
$\dot{m}$ = ablation rate, lb/ft²-sec
Pr = Prandtl number, $\bar{c}_p\mu/k$
q = heat flux, Btu/ft²-sec
T = temperature, °R
V = freestream velocity, fps
ρ = density, lbm/ft³
μ = viscosity, lbm/ft-sec

Subscripts

e = edge of boundary layer
w = wall
D = dissociation

Introduction

MUCH ablation test work is carried out in subsonic air arc-tunnel facilities under conditions attempting to simulate those that exist behind a re-entry shock wave. The usual objective of such tests is to obtain an effective heat of ablation for the purposes of re-entry heat shield design. This effective heat of ablation is based on the rate at which a material ablates when subjected to a certain heat flux:

$$H^* = q/\dot{m} \tag{1}$$

The effective heat of ablation is thus as dependent on the heat flux measurement as on the rate of ablation.

In an air arc tunnel, this flux often is measured by immersing a flat-faced, water-cooled calorimeter into the dissociated air stream. If the air is in thermodynamic equilibrium, the calorimeter will measure essentially the maximum heat that can be transferred to a cold body under the particular conditions of the test. If the air is not in thermodynamic equilibrium, the measured heat flux will be something less than this maximum, depending on both the degree of departure from equilibrium and the catalytic activity of the calorimeter wall.

A measure of this departure can be obtained by subjecting the data to various theoretical models. Using the modified Fay-Riddell equation[1, 2] as a basis for these models, one has for model 1 (equilibrium boundary layer)

$$q = \frac{0.793}{Pr_w^{0.6}} (\rho_e\mu_e)^{0.44}(\rho_w\mu_w)^{0.06} (h_e - h_w) \times \left[1 + (Le^{0.52} - 1)\frac{h_D}{h_e}\right]\left(\frac{\pi V}{2D}\right)^{1/2} \tag{2}$$

for model 2 (frozen flow and a fully catalytic wall) an identical equation but with the Lewis number raised to the 0.63 power, and for model 3 (frozen flow and a noncatalytic wall) an equation identical to model 2 but where the wall enthalpy includes the dissociation enthalpy. In all three models, the last term, the velocity gradient, is that corresponding to a flat face in incompressible flow as given by Truitt.[3]

Experimental Data and Derived Parameters

Having these three equations, a knowledge of the calorimeter wall temperature, freestream enthalpy, pressure, and velocity allows calculating the other data necessary to check measured heat fluxes against those predicted by the three models. The temperature, density, viscosity, and Lewis number can be estimated from the work of Hansen.[4] The degree of dissociation can be estimated from that of Gilmore.[5] The dissociation enthalpy then can be calculated from the heats of dissociation of nitrogen and oxygen at 14,400 Btu/lbm and 6600 Btu/lbm, respectively. Resulting properties corresponding to the outer edge of the boundary layer are given in Table 1.

Properties at the wall are calculated for a temperature of 660°R and a pressure equal to that at the outer edge of the boundary layer. Viscosity is taken as 1.3×10^{-5} lbm/ft-sec and the Prandtl number as 0.73. For model 3, the dissociation enthalpy is added to the 200-Btu/lbm-wall enthalpy of that used in the first two cases.

Heat fluxes are calculated from Table 1, and those actually measured are given in Table 2. The difference between models 1 and 2 is so slight that in Fig. 1 only points for models 1 and 3 are plotted.

Results

The most striking result is the superior correlation obtained with the assumption of frozen flow and a noncatalytic wall. Whereas the assumptions of 1) an equilibrium boundary layer or 2) frozen flow with a fully catalytic wall lead to large scatter and consistently high results, the assumption of frozen flow and a noncatalytic wall is essentially exact at lower densities and degree of dissociation and remains preferable at higher densities. At higher densities, recombination does occur but not to the extent often assumed.

The fact that the copper calorimeter acted noncatalytically is surprising, since metals and metal oxides often are considered to be quite catalytic. If, however, a copper calorimeter

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Received February 18, 1963.

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Table 1 Air properties at outer edge of boundary layer

No.	h_e	P_e	V	T_e	ρ_e	μ_e	Le	h_D
1	8800	2.0	1800	10,600	3.5×10^{-4}	8.3×10^{-5}	0.85	5000
2	6800	2.0	1800	10,000	4.0	7.8	0.97	3600
3	5100	2.0	1900	9,100	4.7	7.2	1.09	2400
4	8700	4.1	2000	10,700	7.1	8.4	0.87	4200
5	5800	5.8	1500	9,200	13.5	7.2	1.10	2000
6	2800	6.2	1900	6,300	23.6	5.6	1.35	900
7	5100	10.0	1800	9,400	22.9	7.4	1.10	1500

Table 2 Measured and calculated heat fluxes

		Calculated by model		
No.	Measured	1	2	3
1	80	185	183	77
2	73	154	154	70
3	66	123	124	63
4	170	291	288	146
5	153	220	220	141
6	90	131	133	87
7	228	271	274	190

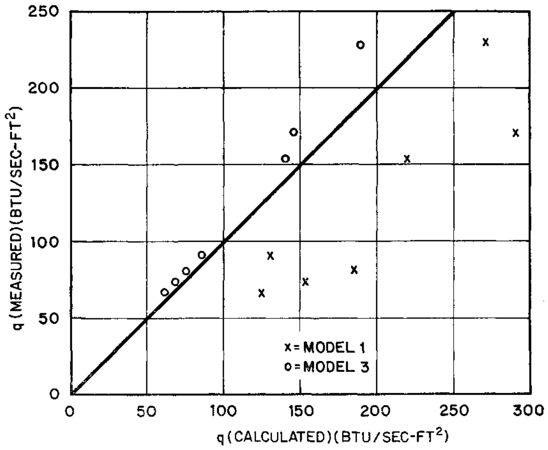

Fig. 1 Comparison of measured and calculated heat rates for two models

is coated with a material such as SiO, catalytic activity is reduced remarkably. Winkler and Griffin[6] have reported that, in a 9000-Btu/lbm dissociated nitrogen stream, a calorimeter coated with SiO decreased heat transfer by 50% over that of a polished copper cylinder. Mavroyannis and Winkler,[7] at freestream densities comparable to those given here, report that, although copper appeared to be catalytic to the recombination of atomic oxygen and nitrogen, oxide-coated copper was only partially catalytic.

In the present case, heat flux to the copper calorimeter was measured in an arc facility that employed carbon electrodes, some of the carbon subliming and reacting with oxygen of the air to form CO. Since it is known that copper oxide strongly adsorbs a surface layer of CO,[8] it is hypothesized that the calorimeter had an oxide layer on its surface and that either this layer was itself noncatalytic to recombination or the strong adsorption of CO rendered it noncatalytic. Work in this area is continuing in our laboratories.

The possibility that a catalytic surface can be rendered so easily noncatalytic makes it imperative that calorimeter-catalytic activity be determined for each test series. Only in this way can there be assurance that the effective heats of ablation calculated from the measured heat rates are realistic. It is strongly suspected that this possible variation in surface catalytic activity often may be responsible for the wide variations reported in effective heats of ablation.

Conclusions

Heat flux measurements are as important in determining effective heats of ablation as are the ablation rates themselves. The fact that flow appears to remain substantially frozen at relatively high pressures in the present experiments makes it imperative that calorimeter poisoning be held to a minimum and catalytic activity towards atomic recombination be determined if accurate and reproducible effective heats of ablation are to be obtained.

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Debye-Huckel Plasma Corrections

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The physical model recently used by McGee and Heller to compute the thermodynamic properties of gaseous plasma is questioned. For the medium-density plasma, the Debye-Huckel theory of point charges is adequate and provides both a cutoff for the electronic partition function and a lowering of the ionization potential. The difficulties of the high-pressure plasma model are mentioned briefly.

In a recent paper, McGee and Heller[1] have computed the thermodynamic properties of the first three elements treated as plasmas in the pressure and temperature ranges 0.0001 to 100 atm and 2000° to 50,000°K, respectively. The physical model that they chose was as follows:

1) The plasma consisted of atoms, ions and, electrons, all of which obeyed classical (Boltzmann) statistics.

2) Neutral gas thermal imperfections were neglected.

Received March 8, 1963. This work was financed by D. S. I. R. through a contract from the Aero. Division of the National Physics Laboratory, England.

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